What you'll Learn About
How to build a polynomial using derivatives

$$\boxed{P(x) = \frac{P(0)x^0}{0!} + \frac{P'(0)x^1}{1!} + \frac{P''(0)x^2}{2!} + \frac{P'''(0)x^3}{3!} + \dots \frac{P^n(0)x^n}{n!}}$$

Center

$P(0) = 7$
$P'(0) = 3$
$P''(0) = 9$
$P'''(0) = 15$
$P^4(0) = 6$
$P^5(0) = 4$
$P^6(0) = 12$

Given the values of the following, construct the 6th degree Taylor Polynomial centered at x = 0

$P(x) = \frac{7x^0}{0!} + \frac{3x^1}{1!} + \frac{9}{2}x^2 + \frac{15}{6}x^3 + \frac{6}{24}x^4 + \frac{4}{120}x^5 + \frac{12}{720}x^6 + \frac{22}{7!}x^7 + \frac{50}{8!}x^8$

$P(x) = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6$

$P(0) = a$
$\boxed{7 = a}$

$P'(x) = b + 2cx + 3dx^2 + 4ex^3 + 5fx^4 + 6gx^5$
$P'(0) = b$
$\boxed{3 = b}$

$P''(x) = 2c + 6dx + 12ex^2 + 20fx^3 + 30gx^4$
$P''(0) = 2c$
$9 = 2c$ $\quad \boxed{c = \frac{9}{2}}$

$P'''(x) = \cancel{2} + 6d + 24ex + 60fx^2 + 120gx^3$
$P'''(x) = \cancel{}\ 6d \quad 15 = 6d \quad \boxed{d = 15/6}$

$P^4(x) = 24e + 120fx + 360gx^2 \quad 6 = 24e \quad \boxed{e = 6/24}$

$P^5(x) = 120f + 720gx \quad 4 = 120f \quad \boxed{f = 4/120}$

$P^6(x) = 720g \quad 12 = 720g \quad \boxed{g = 12/720}$

What would be the next 2 terms if $P^7(0) = 22$ and $P^8(0) = 50$?

Third Order → 3rd derivative

x^4

Approximate $f(2)$ $P(2) =$	Given the values of the following, construct the 4th degree Taylor Polynomial centered at x = 0 1. $P(0)=2$ $P'(0)=5$ $P''(0)=8$ $P'''(0)=11$ $P^4(0)=14$ $P(x-0) =$ $P(x) = \frac{2x^0}{0!} + \frac{5x^1}{1!} + \frac{8x^2}{2!} + \frac{11x^3}{3!} + \frac{14x^4}{4!}$ $P(x) = 2 + 5x + 4x^2 + \frac{11}{6}x^3 + \frac{7}{12}x^4$ 2. $P(0)=5$ $P'(0)=-2$ $P''(0)=7$ $P'''(0)=-4$ $P^4(0)=10$ Given the values of the following, construct the 4th degree Taylor Polynomial centered at x = 2 3. $P(2)=2$ $P'(2)=5$ $P''(2)=8$ $P'''(2)=11$ $P^4(2)=14$ $P(x-2) = \frac{2(x-2)^0}{0!} + \frac{5(x-2)^1}{1!} + \frac{8(x-2)^2}{2!} + \frac{11(x-2)^3}{3!} + \frac{14(x-2)^4}{4!}$ Given the values of the following, construct the 4th degree Taylor Polynomial centered at x = -2 4. $P(-2)=5$ $P'(-2)=-2$ $P''(-2)=7$ $P'''(-2)=-4$ $P^4(-2)=10$

CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Watts and Kennedy

Chapter 9: MaClaurin Series

What you'll Learn About
How to write terms given a power series
How to take the derivative and anti-derivative of a power series
Identifying important types of power series

$f(x) = \frac{1^{st}}{1-r}$

① Taking Derivatives
② Plug in Center
③ Build the polynomial

center at $x=0$

1a. Build the MaClaurin Series for $f(x) = \dfrac{1}{1-x}$

$P(x) = \frac{1x^0}{0!} + \frac{1x^1}{1!} + \frac{2x^2}{2!} + \frac{6x^3}{3!} + \dots$

$f(x) = \frac{1}{1-x} = (1-x)^{-1}$ $\quad f(0)=1$

$P(x) = 1 + x + x^2 + x^3 + x^4 + \dots$

$f'(x) = +(1-x)^{-2} = \frac{1}{(1-x)^2}$ $\quad f'(0)=1$

$f''(x) = +2(1-x)^{-3} = \frac{2}{(1-x)^3}$ $\quad f''(0)=2$

$P(x) = \sum_{n=0}^{\infty} x^n$

$f'''(x) = +6(1-x)^{-4} = \frac{6}{(1-x)^4}$ $\quad f'''(0)=6$

Geometric
$r = x$

b. Determine the Interval of Convergence of the series.

$-1 < x < 1$

c. Take the derivative of the power series for $f(x) = \dfrac{1}{1-x}$

$P(x) = \sum_{n=0}^{\infty} x^n$ $\qquad P'(x) = \sum_{n=1}^{\infty} n x^{n-1}$

Terms $= 1 + x + x^2 + x^3$ $\qquad$ Terms $= 0 + \underset{n=1}{1} + \underset{n=2}{2x} + \underset{n=3}{3x^2}$

d. Take the anti-derivative of the power series for $f(x) = \dfrac{1}{1-x}$

$P(x) = \sum_{n=0}^{\infty} x^n$ $\qquad \int P(x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$

Terms $= 1 + x + x^2 + x^3$ $\qquad$ Terms $= x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4$

2a. Build the MaClaurin Series for $f(x) = \ln(1+x)$

$f(x) = \ln(1+x)$

$f'(x) = \frac{1}{1+x} = (1+x)^{-1}$

$f''(x) = -(1+x)^{-2}$

$f'''(x) = 2(1+x)^{-3}$

$f(0) = 0$

$f'(0) = 1$

$f''(0) = -1$

$f'''(0) = 2$

$$P(x) = \frac{0x^0}{0!} + \frac{1x^1}{1!} - \frac{1x^2}{2!} + \frac{2x^3}{3!} = x - \frac{x^2}{2} + \frac{x^3}{3}$$

b. Determine the Interval of Convergence of the series.

c. Take the derivative of the power series for $f(x) = \ln(1+x)$

d. Take the anti-derivative of the power series for $f(x) = \ln(1+x)$